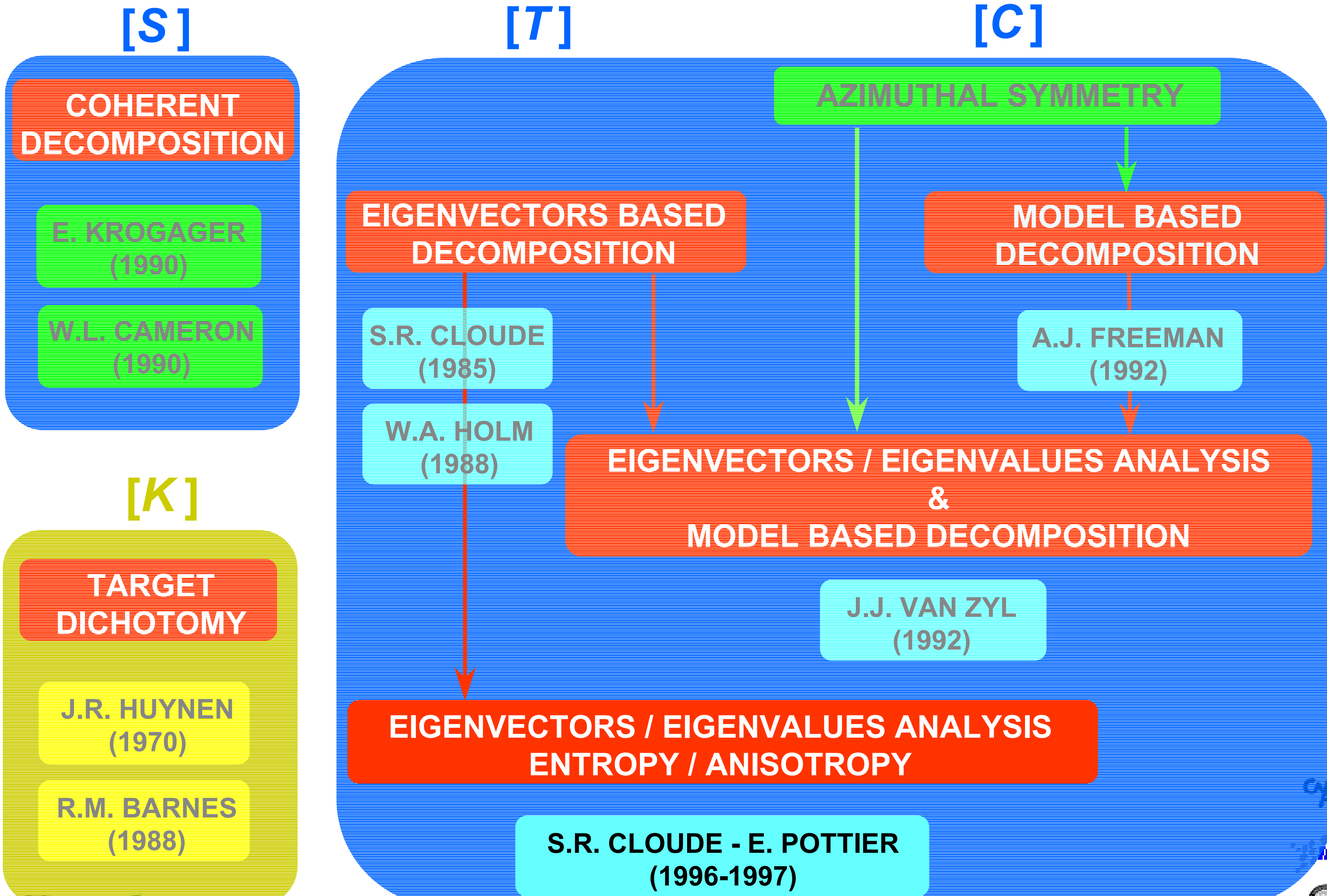
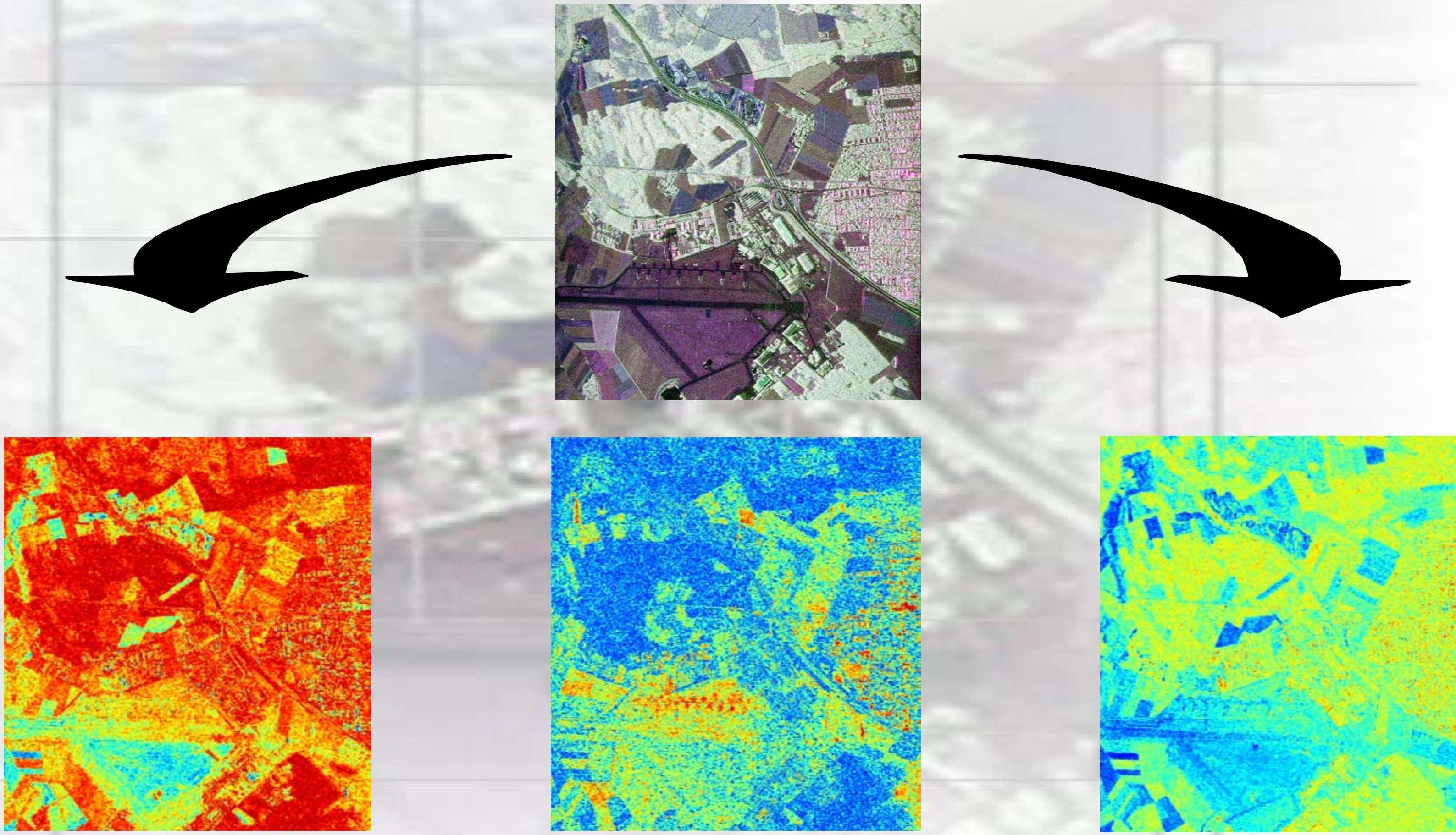
DECOMPOSITION THEOREMS



THE H / A / $\underline{\alpha}$ POLARIMETRIC TARGET DECOMPOSITION THEOREM



S.R. CLOUDE - E. POTTIER (1995 - 1996)

H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle[T]\rangle = \frac{1}{N}\sum_{i=1}^{N}\underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N}[T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle[T]\rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3}\lambda_k}$$

H / A / α DECOMPOSITION

$$\langle[T]\rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

PARAMETERISATION OF THE SU(3) UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3) \\ \sin(\alpha_1)\cos(\beta_1)e^{j\delta_1} & \sin(\alpha_2)\cos(\beta_2)e^{j\delta_2} & \sin(\alpha_3)\cos(\beta_3)e^{j\delta_3} \\ \sin(\alpha_1)\sin(\beta_1)e^{j\gamma_1} & \sin(\alpha_2)\sin(\beta_2)e^{j\gamma_2} & \sin(\alpha_3)\sin(\beta_3)e^{j\gamma_3} \end{bmatrix}$$

TARGET 1 TARGET 2 TARGET 3

H / A / $\underline{\alpha}$ DECOMPOSITION

PROBABILITIES

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

AVERAGED PARAMETERS

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \qquad \underline{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$

$$\underline{\gamma} = P_1\gamma_1 + P_2\gamma_2 + P_3\gamma_3 \qquad \underline{\delta} = P_1\delta_1 + P_2\delta_2 + P_3\delta_3$$

UNITARY TARGET VECTOR ($\underline{u}_0$) OF THE MEAN DOMINANT MECHANISM

$$\underline{u}_0 = \left[\cos(\underline{\alpha}) \quad \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \quad \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}}\right]^T$$

MEAN SCATTERING MECHANISM

UNITARY VECTOR $\underline{u}_0$

$$\underline{u}_0 = \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

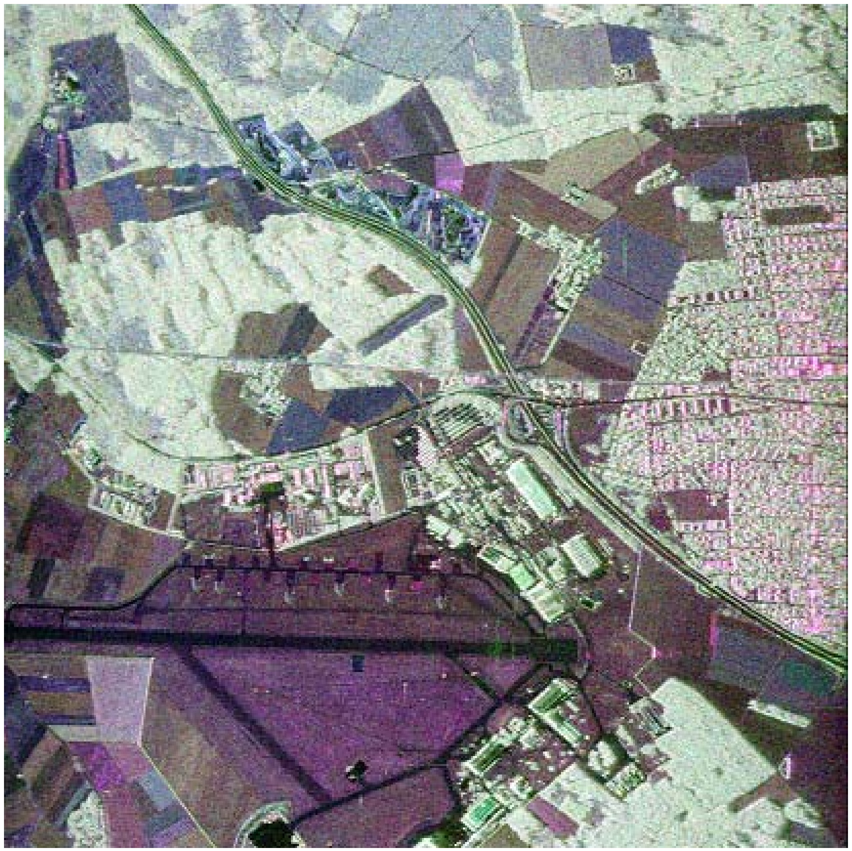
TARGET MAGNITUDE

$$\underline{\lambda} = P_1\lambda_1 + P_2\lambda_2 + P_3\lambda_3 = \frac{\sum_{i=1}^{3}\lambda_i^2}{\sum_{k=1}^{3}\lambda_k}$$

TARGET VECTOR $\underline{k}_0$

$$\underline{k}_0 = \sqrt{\underline{\lambda}}\begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})\,e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})\,e^{j\underline{\gamma}} \end{bmatrix}$$

H / A / α DECOMPOSITION



$2A_0$ $\quad$ $B_0 + B$ $\quad$ $B_0 - B$

$\sqrt{\lambda}\,cos(\alpha)$ $\quad$ $\sqrt{\lambda}\,sin(\alpha)\,cos(\beta)$

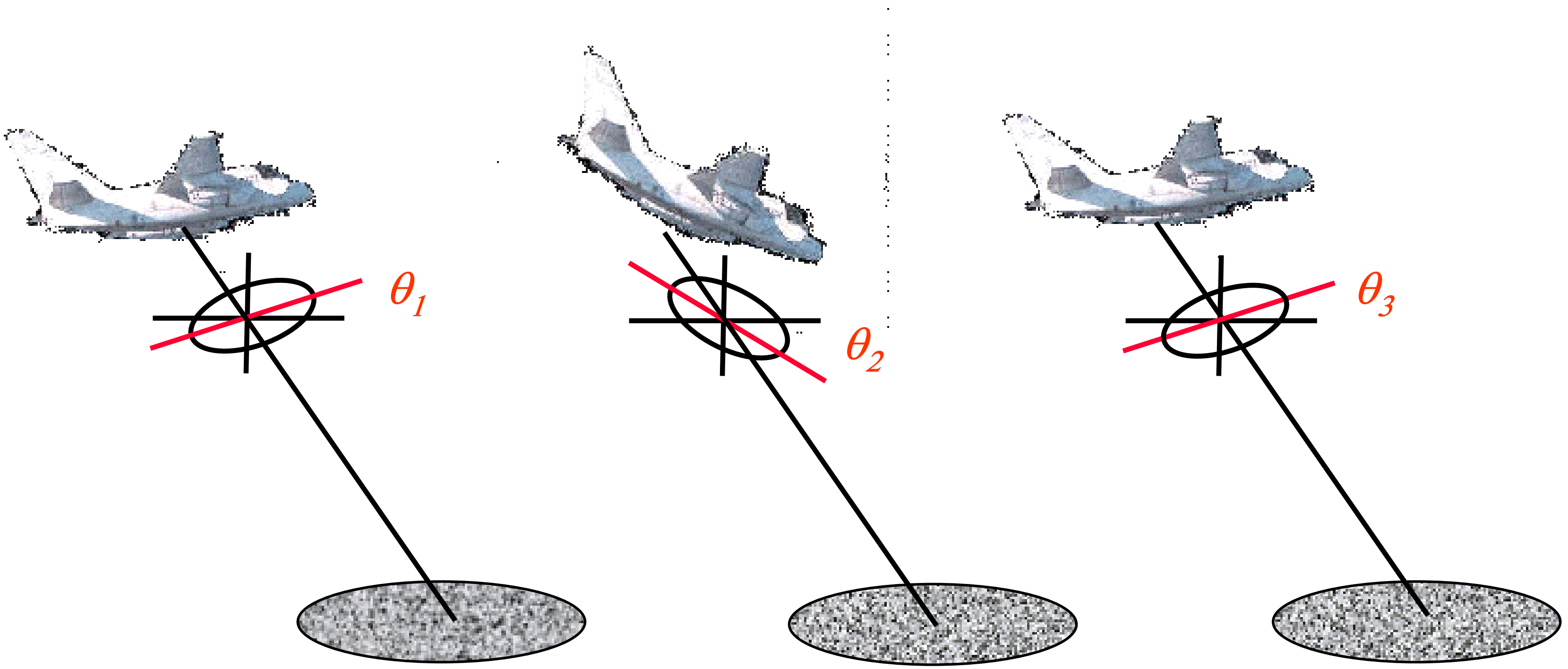
$\sqrt{\lambda}\,sin(\alpha)\,sin(\beta)$

H / A / α DECOMPOSITION

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

$\sqrt{\lambda}\cos(\alpha)$ $\quad \sqrt{\lambda}\sin(\alpha)\cos(\beta)$ $\quad \sqrt{\lambda}\sin(\alpha)\sin(\beta)$

ROLL INVARIANCE PROPERTY



SAME PHYSICAL PHENOMENOUS **WHATEVER**

THE ANTENNA ***ORIENTATION ANGLE***

AROUND THE ***RADAR LINE OF SIGHT***

H / A / α DECOMPOSITION

ROLL INVARIANCE PROPERTY

ORIENTED (θ) COHERENCY MATRIX

$$\langle [T(\theta)] \rangle = [U_R(\theta)] \langle [T] \rangle [U_R(\theta)]^{-1}$$

SU(3) UNITARY ROTATION MATRIX (θ)

$$[U_R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T(\theta)] \rangle = [U_3(\theta)] [\Sigma] [U_3(\theta)]^{-1}$$

EIGENVALUES $\lambda_1 \ \lambda_2 \ \lambda_3$: ROLL INVARIANT

PROBABILITIES $P_1 \ P_2 \ P_3$: ROLL INVARIANT

H / A / α DECOMPOSITION

EIGENVECTORS UNITARY MATRIX

$$[U_3(\theta)] = [U_R(\theta)][U_3]$$

↓

PARAMETERIZATION OF THE UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3) \\ \sin(\alpha_1)\cos(\beta'_1)e^{j\delta'_1} & \sin(\alpha_2)\cos(\beta'_2)e^{j\delta'_2} & \sin(\alpha_3)\cos(\beta'_3)e^{j\delta'_3} \\ \sin(\alpha_1)\sin(\beta'_1)e^{j\gamma'_1} & \sin(\alpha_2)\sin(\beta'_2)e^{j\gamma'_2} & \sin(\alpha_3)\sin(\beta'_3)e^{j\gamma'_3} \end{bmatrix}$$

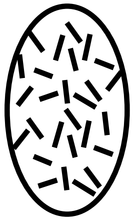
↓

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$: ROLL INVARIANT

PHYSICAL INTERPRETATION

H / A / $\underline{\alpha}$ DECOMPOSITION

ANISOTROPIC PARTICLES CLOUD



$$[S]=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \Rightarrow [T]=\begin{bmatrix} \varepsilon & \mu & 0 \\ \mu^* & \nu & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\langle [T(\theta)] \rangle_\theta = \frac{1}{2}\begin{bmatrix} 2\varepsilon & 0 & 0 \\ 0 & \nu & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

AVERAGING OVER ALL ORIENTATION ANGLES

WITH : $P(\theta)=\frac{1}{2\pi}$

$$\lambda_1=\varepsilon \Rightarrow P_1=\frac{\varepsilon}{(\varepsilon+\nu)}$$

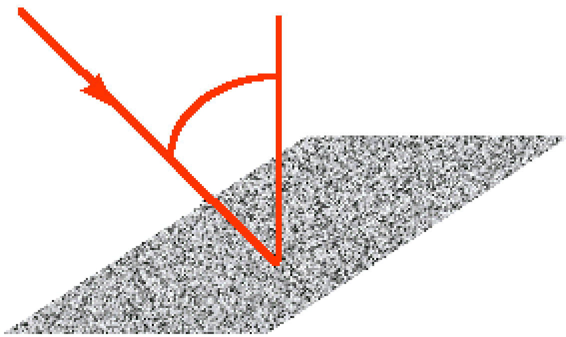
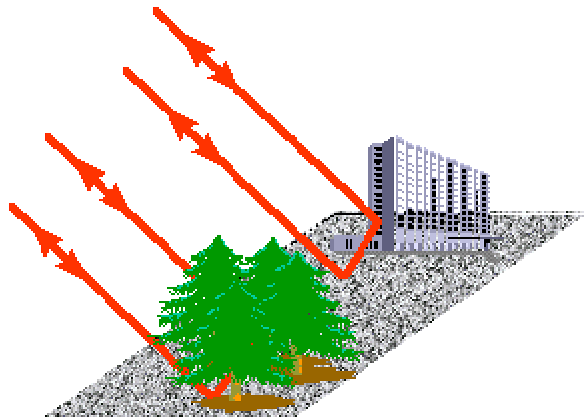
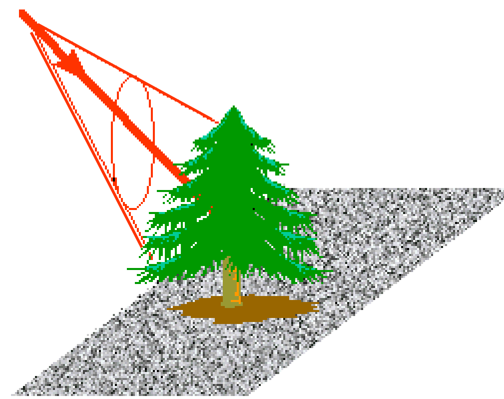
$$\lambda_2=\lambda_3=\frac{\nu}{2} \Rightarrow P_2=P_3=\frac{\nu}{2(\varepsilon+\nu)}$$

$$[U_3]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} \alpha_1=0 \\ \alpha_2=\alpha_3=\frac{\pi}{2} \end{matrix}$$

$$\underline{\alpha}=\alpha_1 P_1+\alpha_2 P_2+\alpha_3 P_3=\frac{\pi}{2}(P_2+P_3)$$

H / A / α DECOMPOSITION

α PHYSICAL INTERPRETATION

SINGLE BOUNCE SCATTERING (ROUGH SURFACE)	DOUBLE BOUNCE SCATTERING	VOLUME SCATTERING
		
$a \mapsto b \Rightarrow \nu \mapsto 0$ $\Downarrow$ $\alpha \mapsto 0$	$a \mapsto -b \Rightarrow \varepsilon \mapsto 0$ $\Downarrow$ $\alpha \mapsto \frac{\pi}{2}$	$a >> b \Rightarrow \varepsilon \approx \nu$ $\Downarrow$ $\alpha \mapsto \frac{\pi}{4}$

H / A / α DECOMPOSITION

EIGENVALUES λ_1 λ_2 λ_3 : ROLL INVARIANT

PROBABILITIES P_1 P_2 P_3 : ROLL INVARIANT

↓

ENTROPY

(DEGREE OF RANDOMNESS
STATISTICAL DISORDER)

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

PURE TARGET

$\lambda_1 = SPAN \quad \lambda_2 = 0 \quad \lambda_3 = 0$

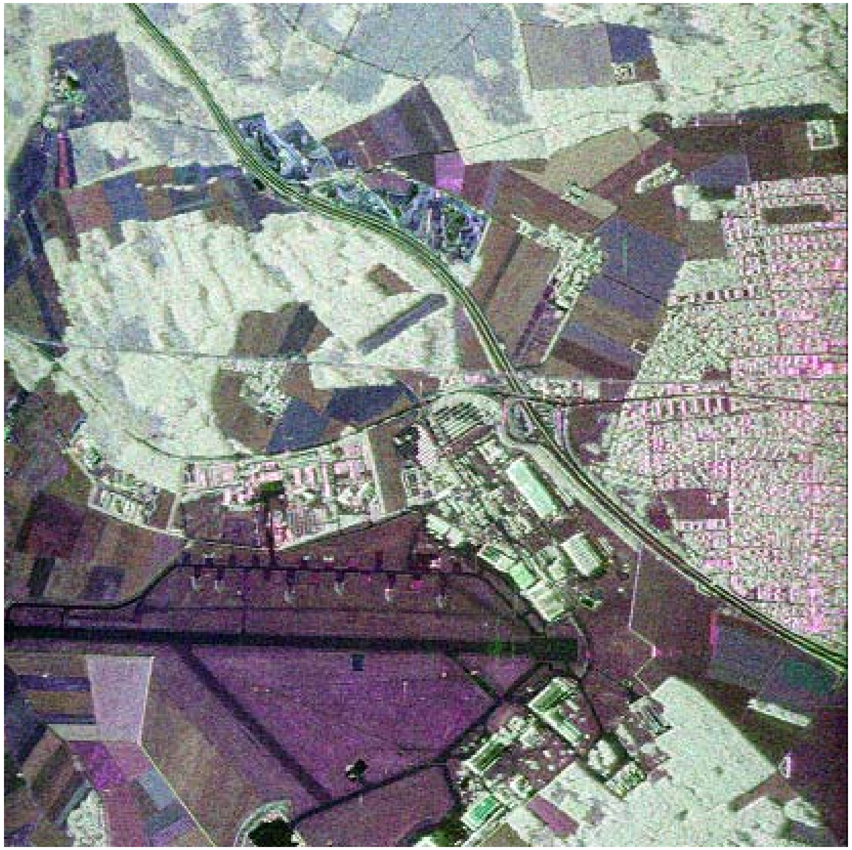
$$H = 0$$

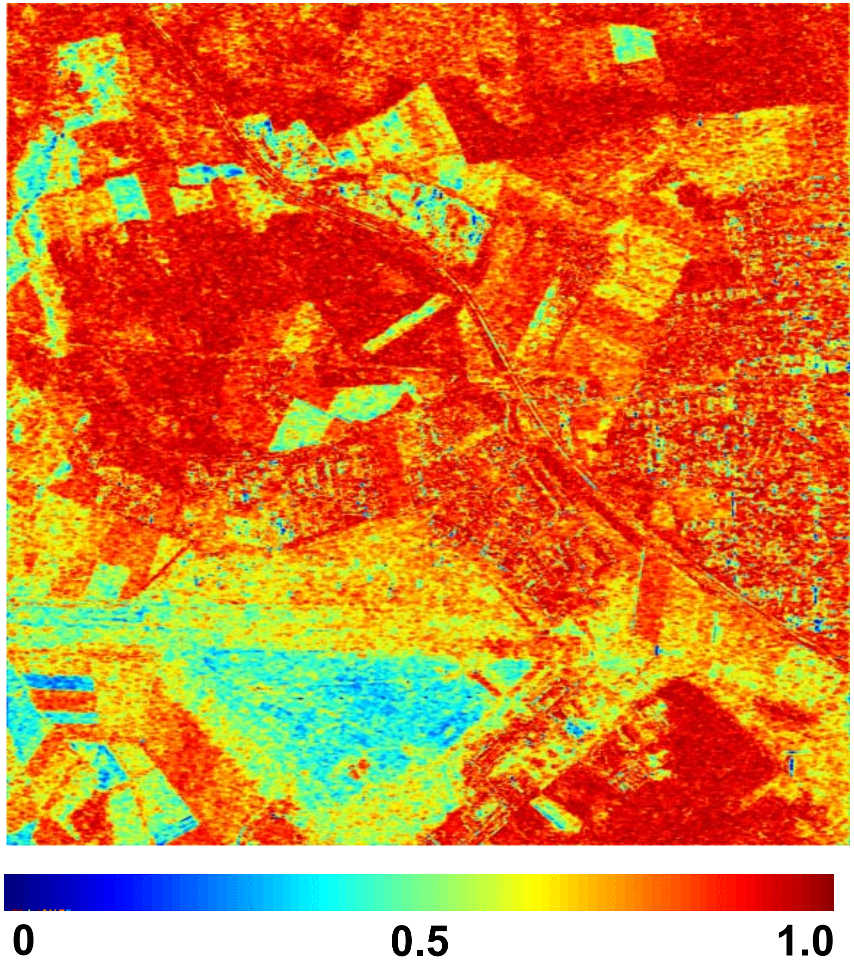
DISTRIBUTED TARGET

$\lambda_1 = \lambda_2 = \lambda_3 = SPAN / 3$

$$H = 1$$

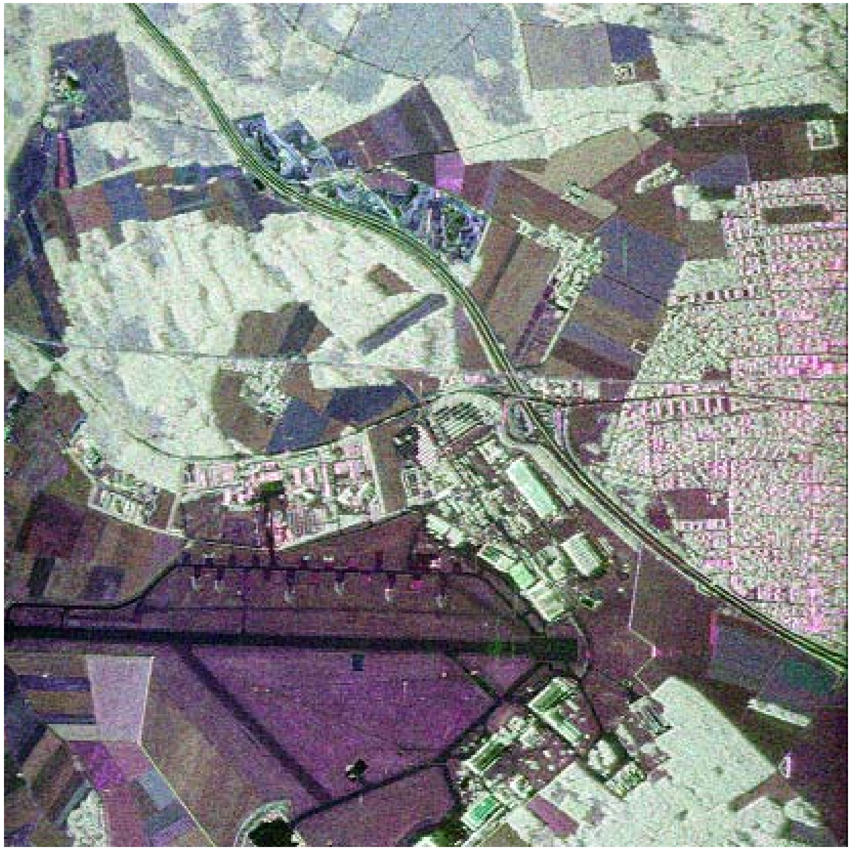
H / A / $\underline{\alpha}$ DECOMPOSITION



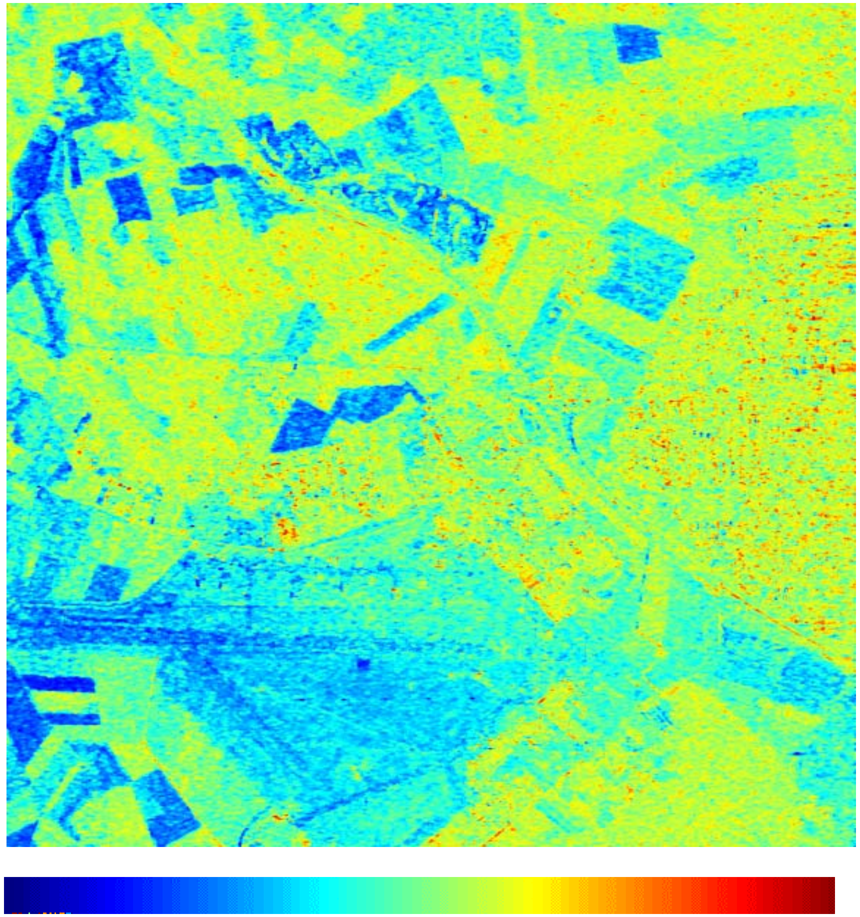


ENTROPY (H)

H / A / α DECOMPOSITION



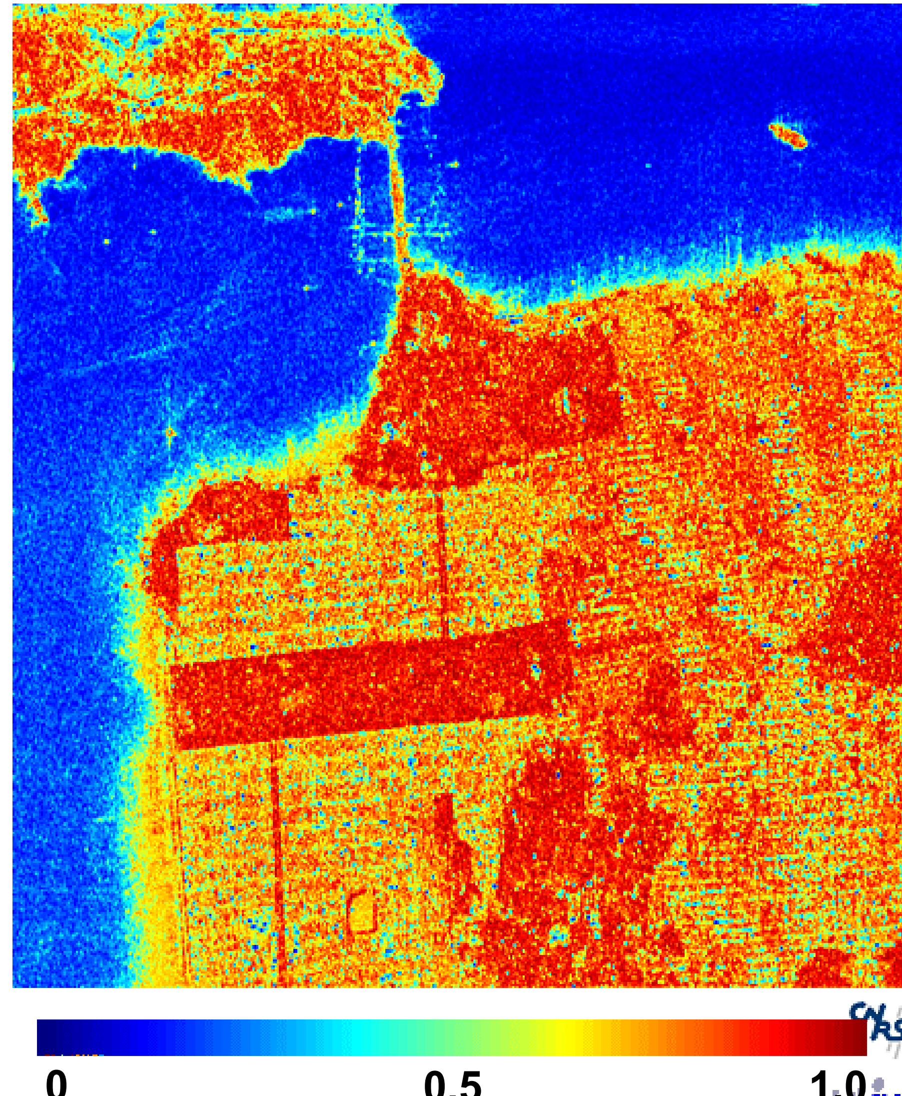
$2A_0$ $B_0 + B$ $B_0 - B$



α PARAMETER

H / A / α DECOMPOSITION

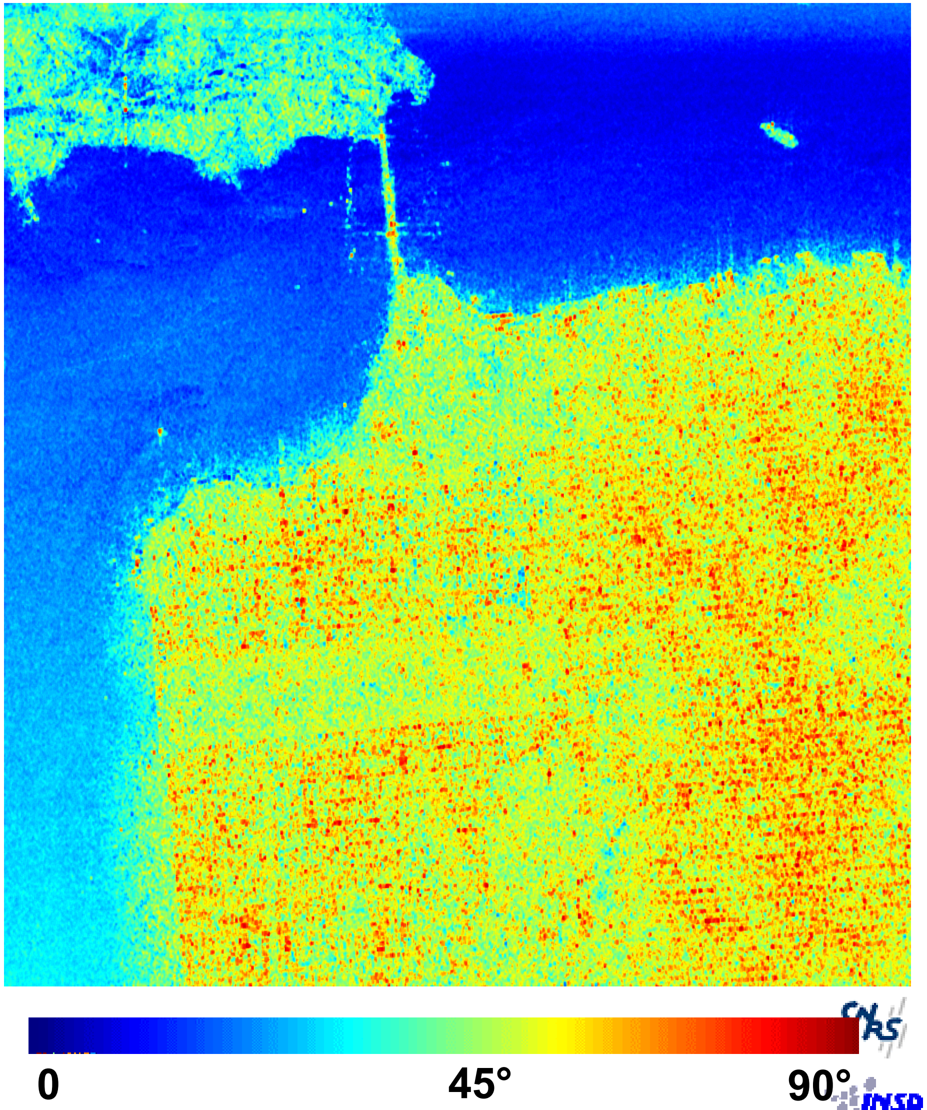
$2A_0$ $B_0 + B$ $B_0 - B$



ENTROPY (H)

H / A / α DECOMPOSITION

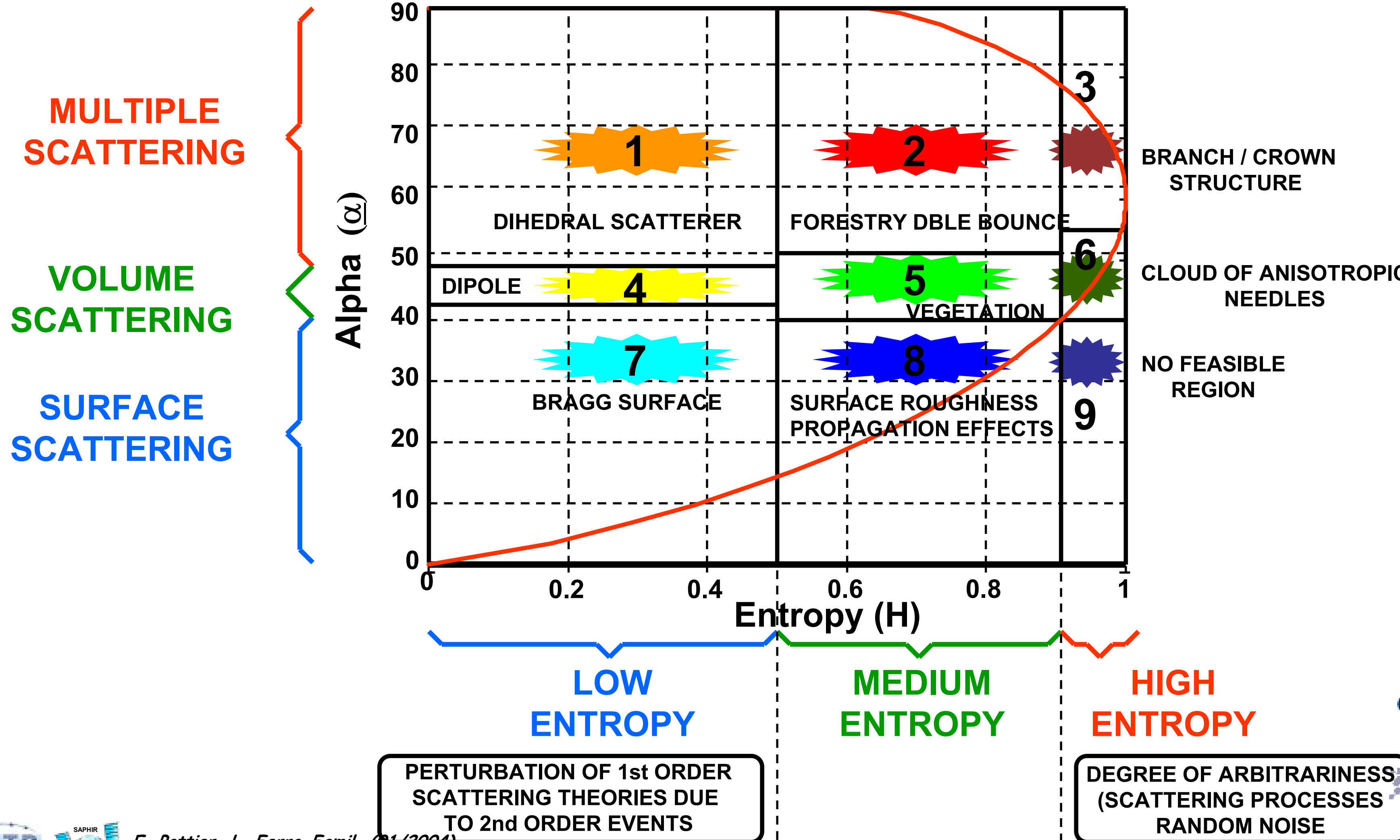
$2A_0$ $B_0 + B$ $B_0 - B$



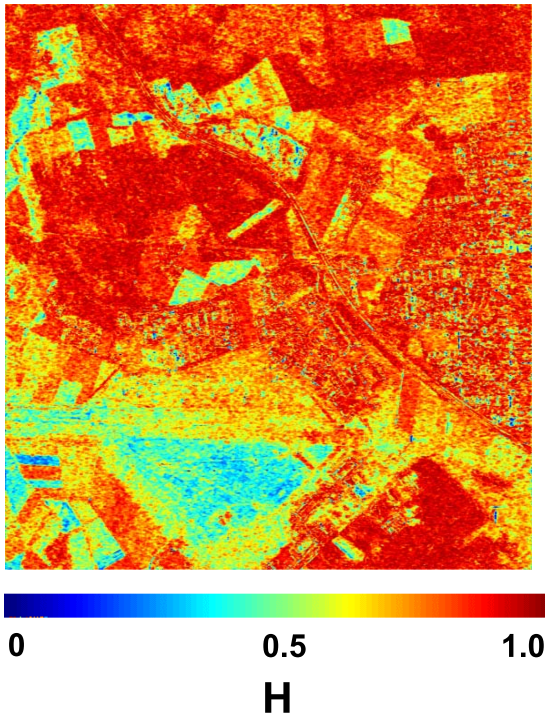
α PARAMETER

H / A / α DECOMPOSITION

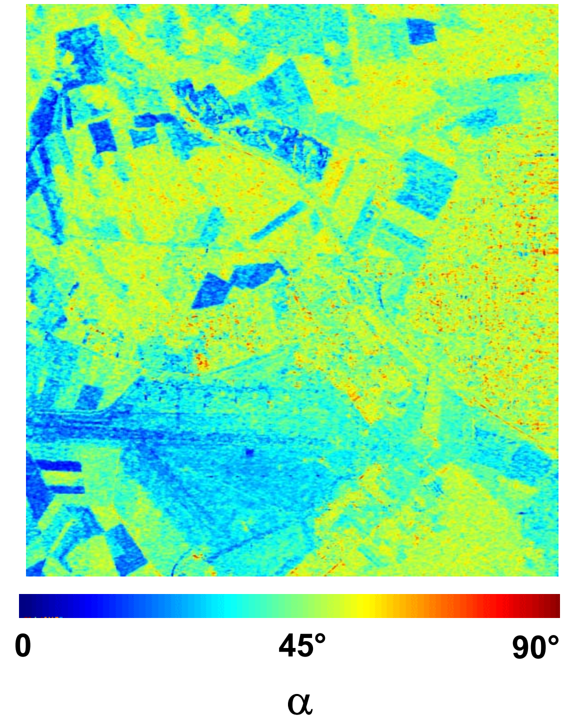
SEGMENTATION OF THE H / α SPACE

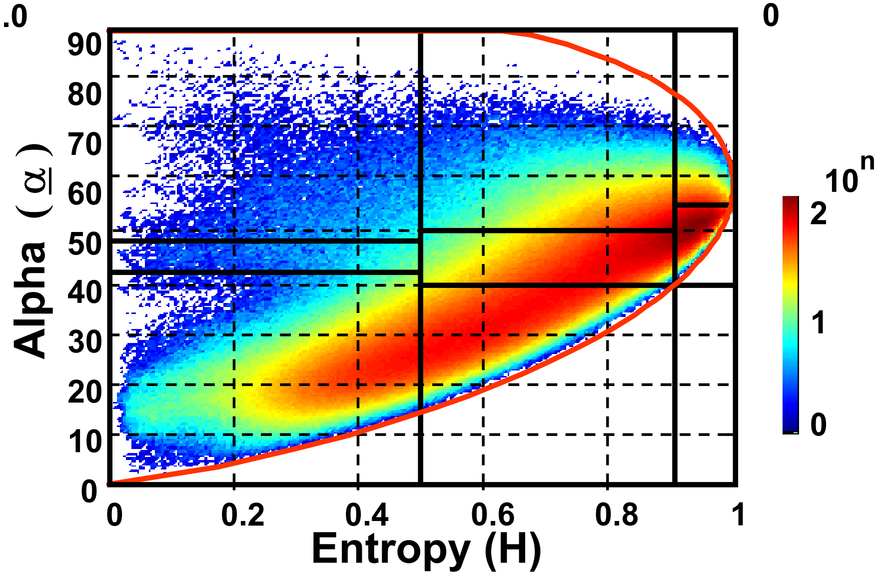


H / A / $\underline{\alpha}$ DECOMPOSITION



POLSAR DATA DISTRIBUTION IN THE H / $\underline{\alpha}$ PLANE





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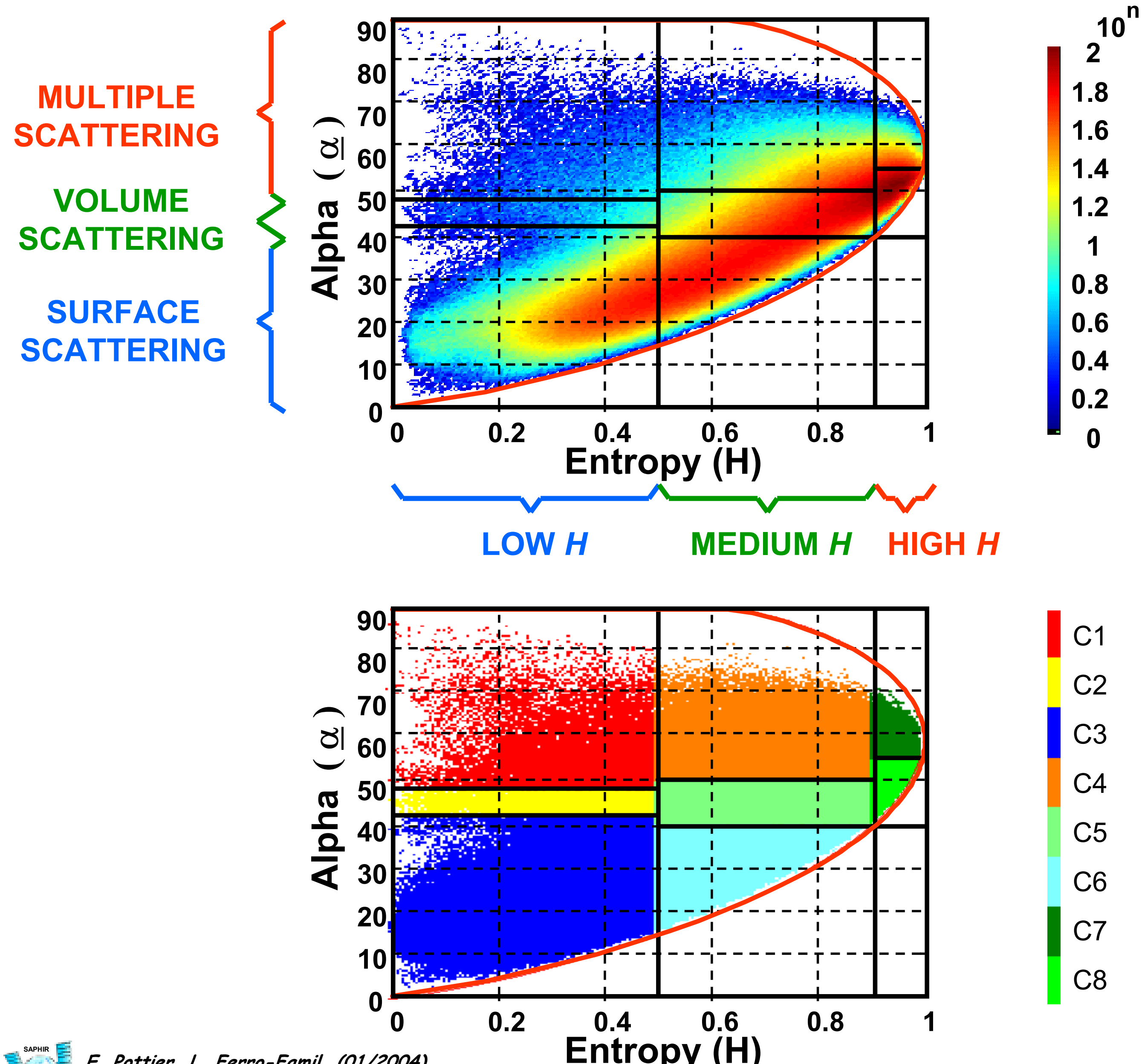
H / A / α DECOMPOSITION



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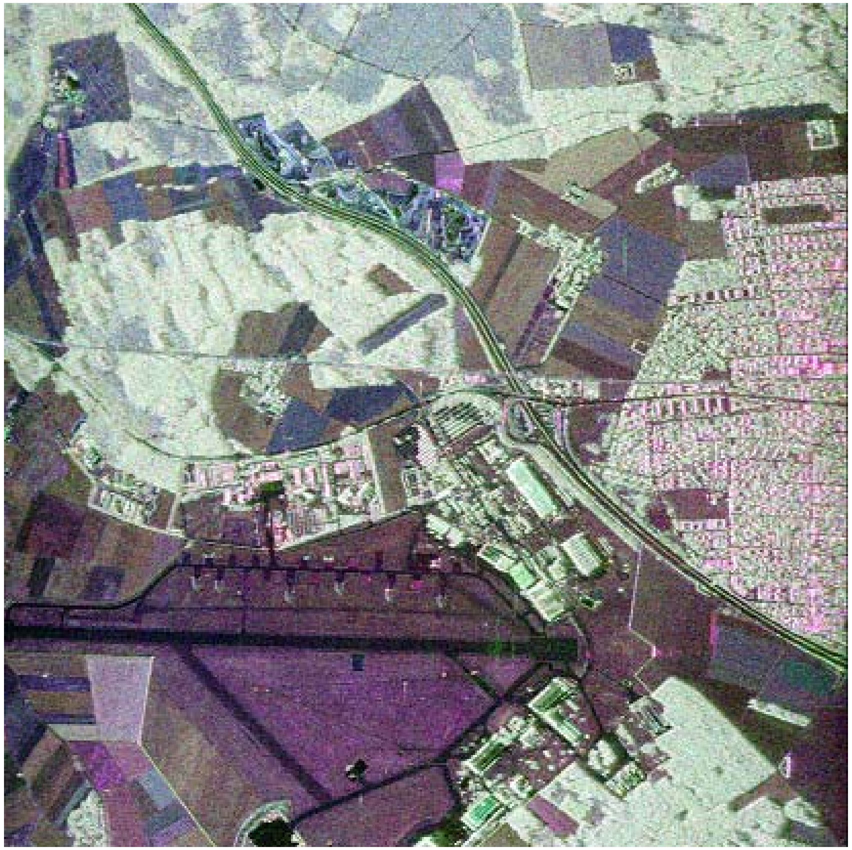
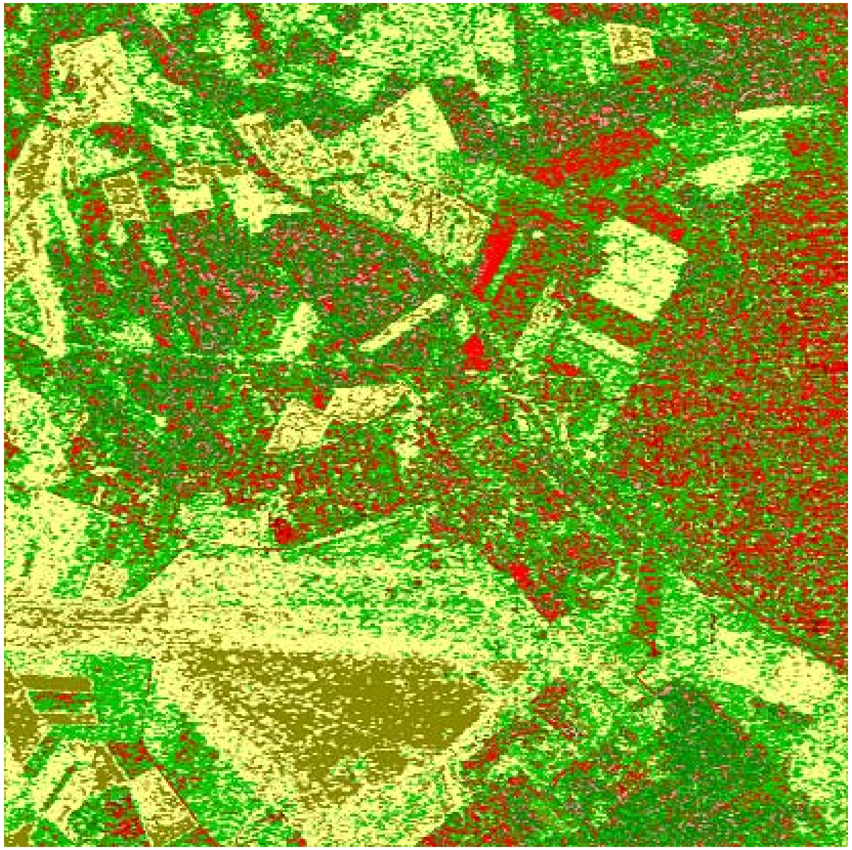
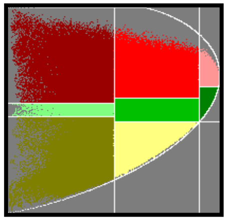
H / A / $\underline{\alpha}$ DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

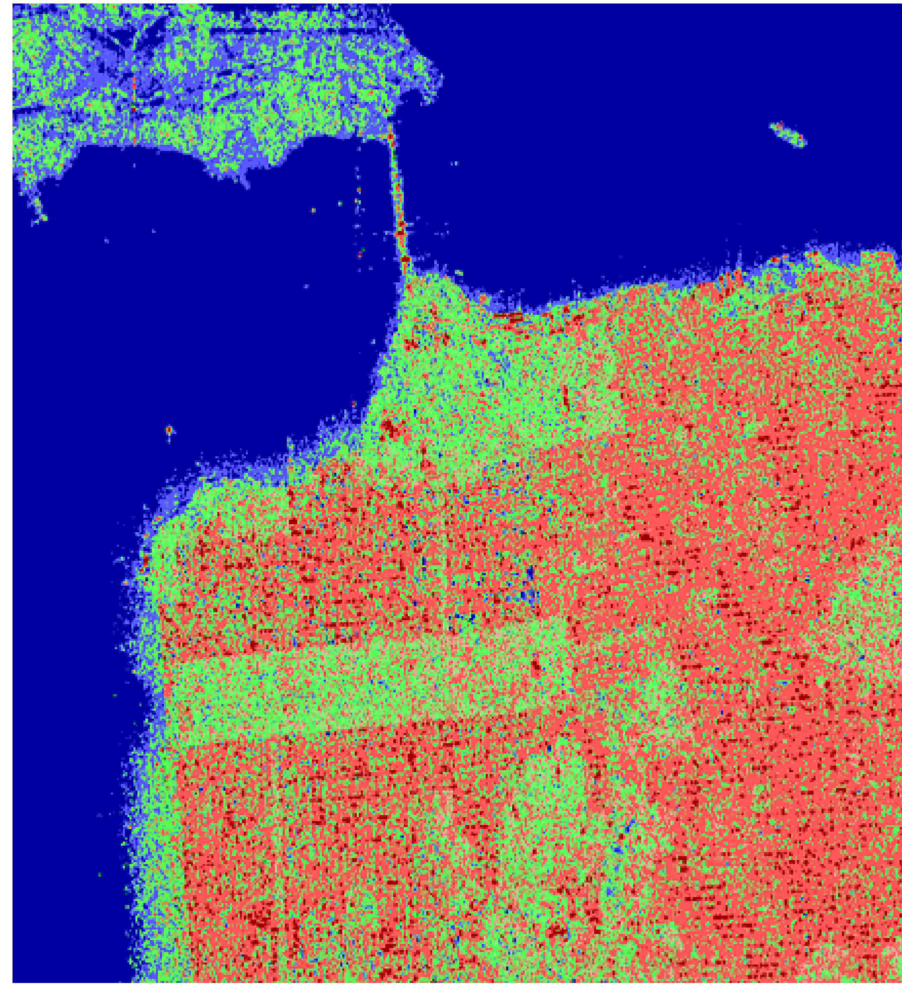
H - $\underline{\alpha}$ classification



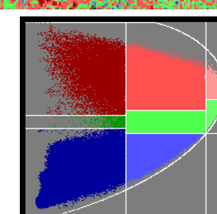


H / A / α DECOMPOSITION

H - α classification



$2A_0$ $\quad$ $B_0 + B$ $\quad$ $B_0 - B$



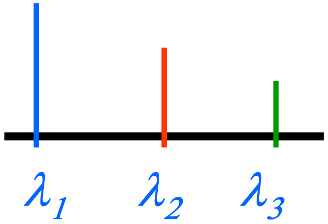
H / A / a DECOMPOSITION

DIFFICULT MECHANISM DISCRIMINATION WHEN : H > 0.7

ANISOTROPY

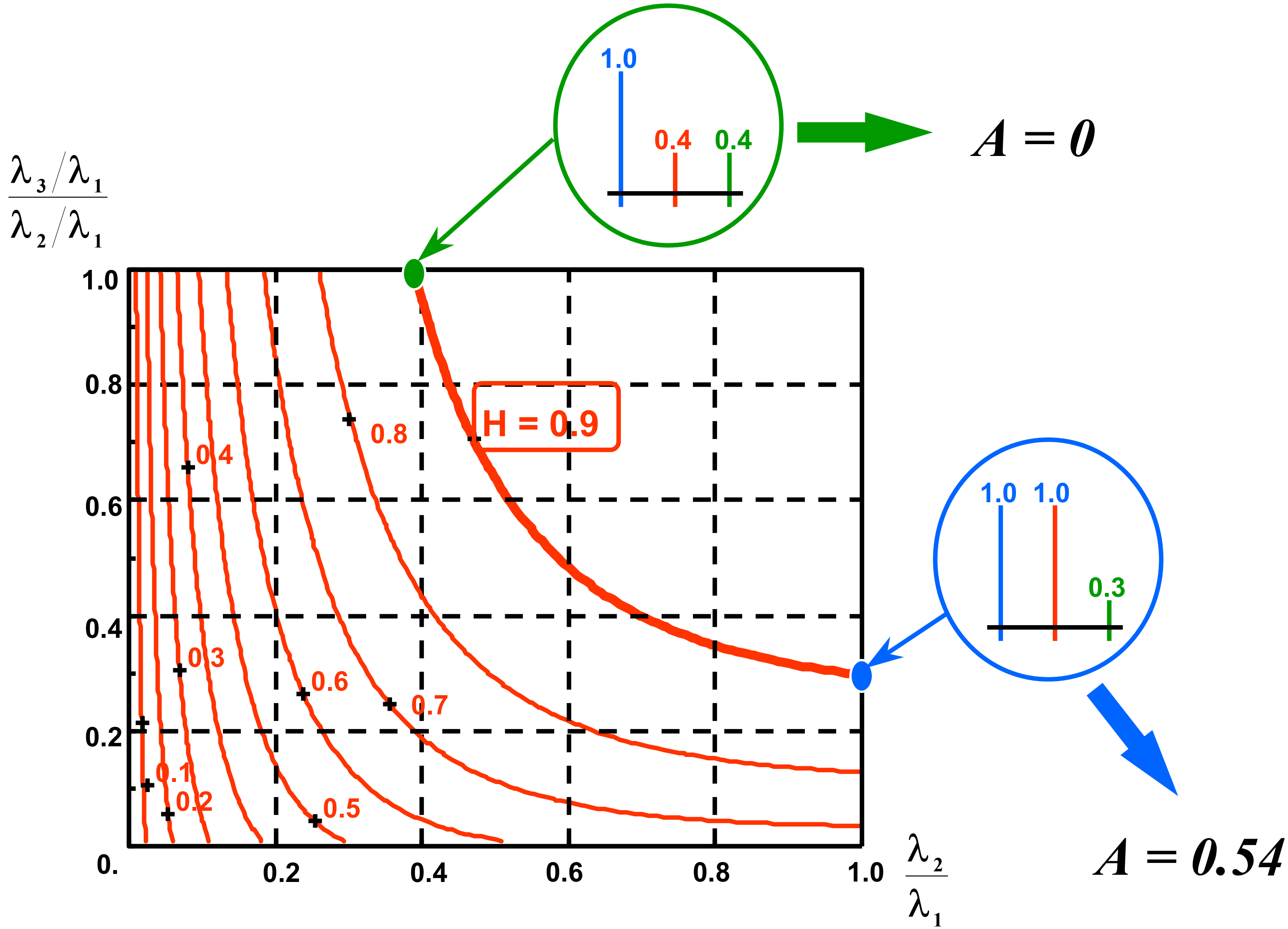
(EIGENVALUES SPECTRUM)

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

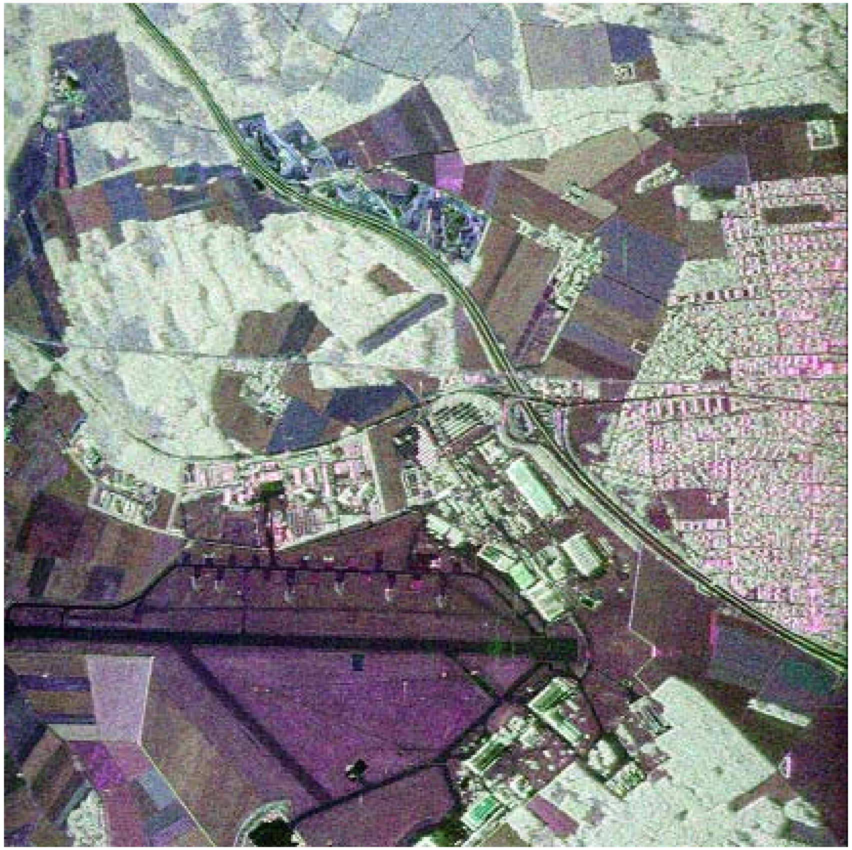


- COMPLEMENTARY TO ENTROPY
- DISCRIMINATION WHEN H > 0.7
- ROLL INVARIANT

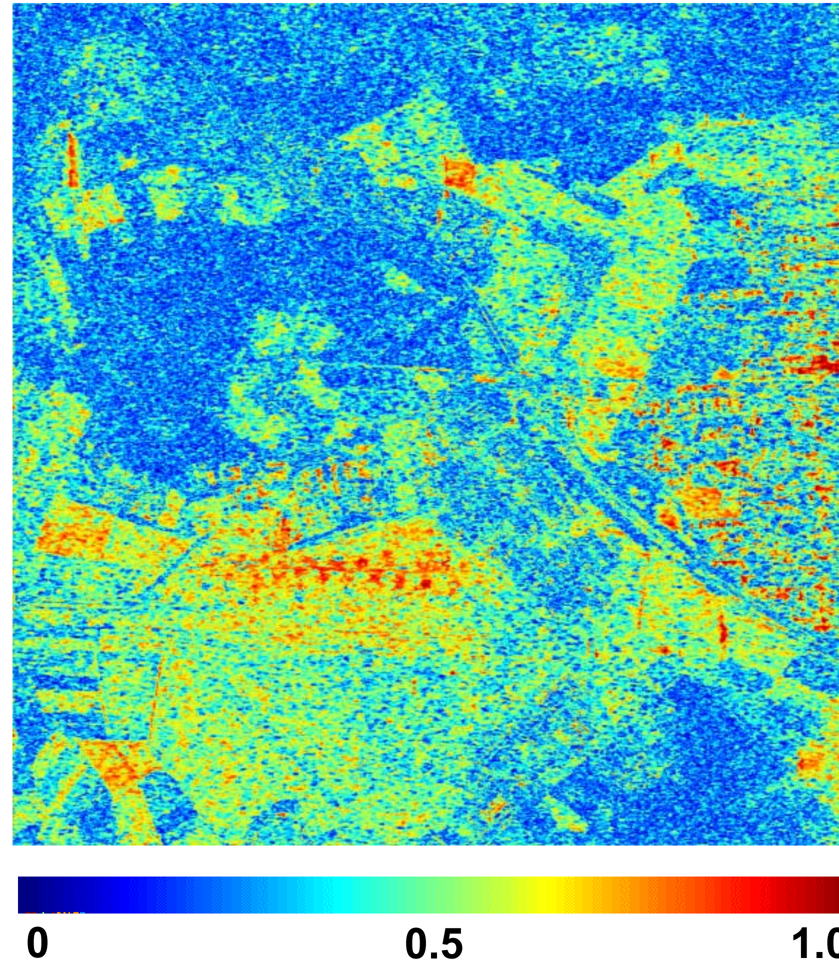
H / A / α DECOMPOSITION



H / A / α DECOMPOSITION

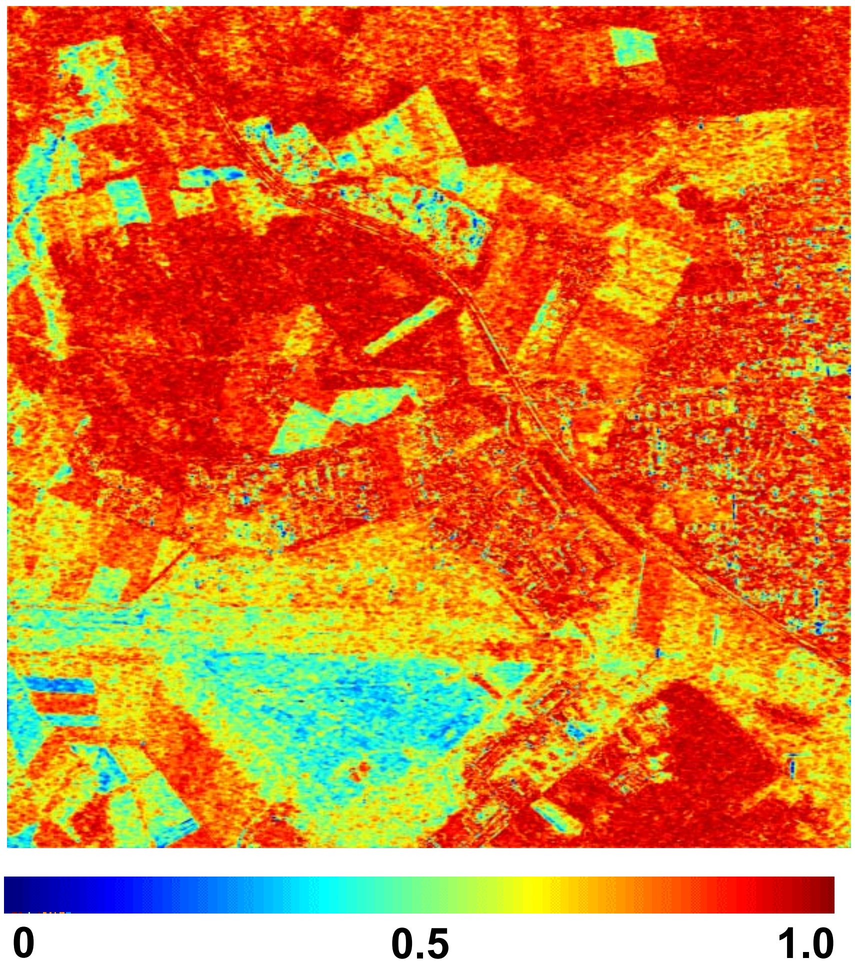


$2A_0$ $B_0 + B$ $B_0 - B$

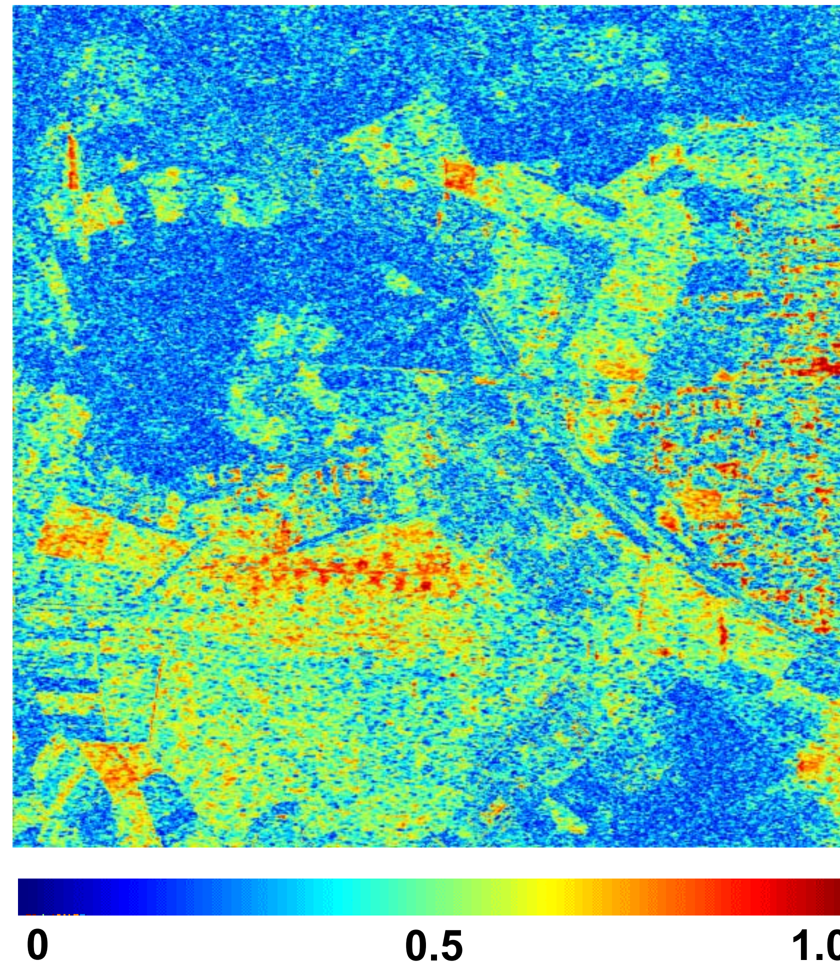


ANISOTROPY (A)

H / A / α DECOMPOSITION



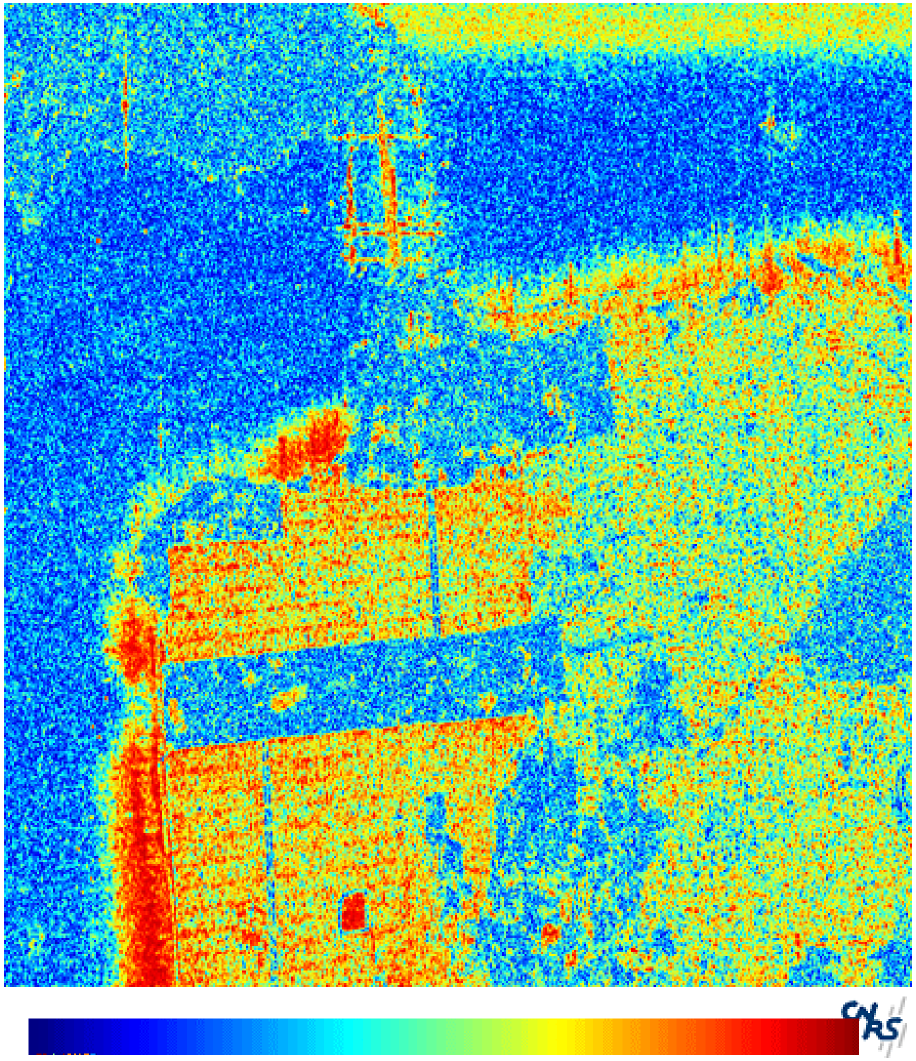
ENTROPY (H)



ANISOTROPY (A)

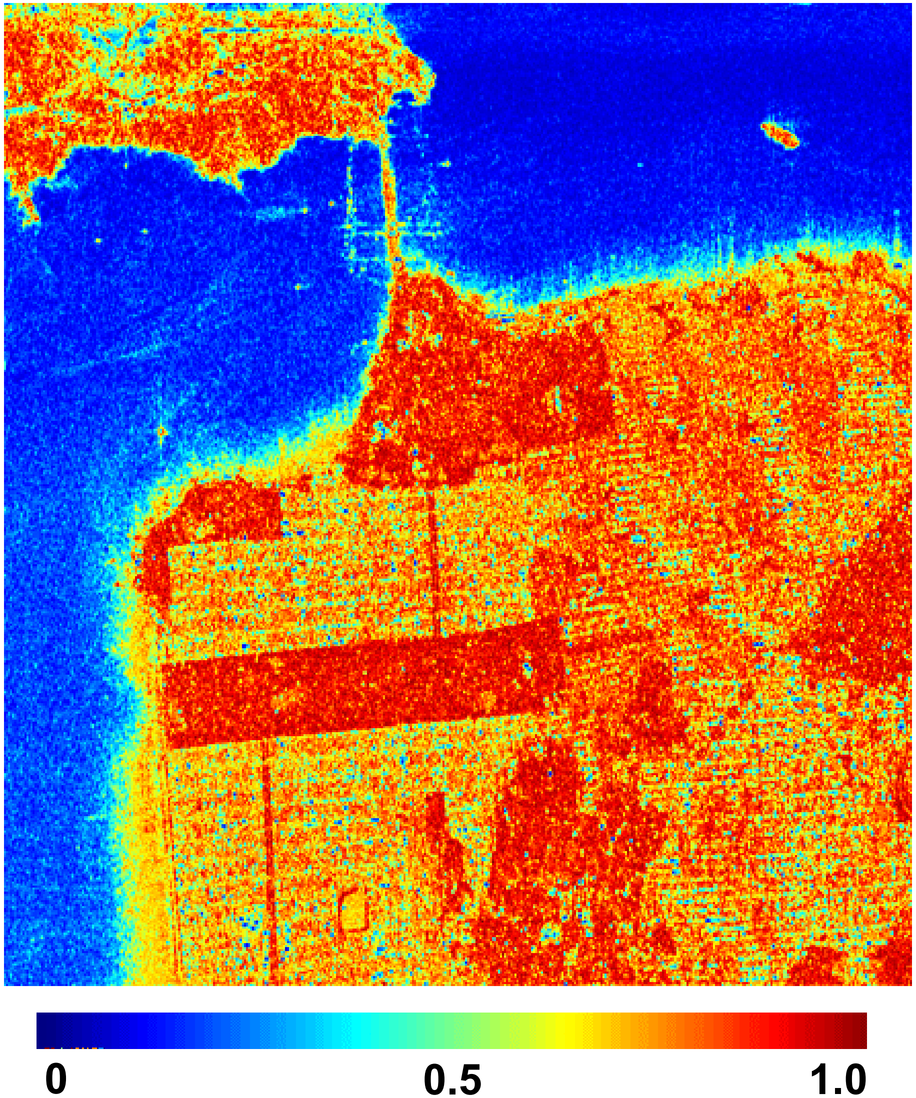
H / A / $\underline{\alpha}$ DECOMPOSITION

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

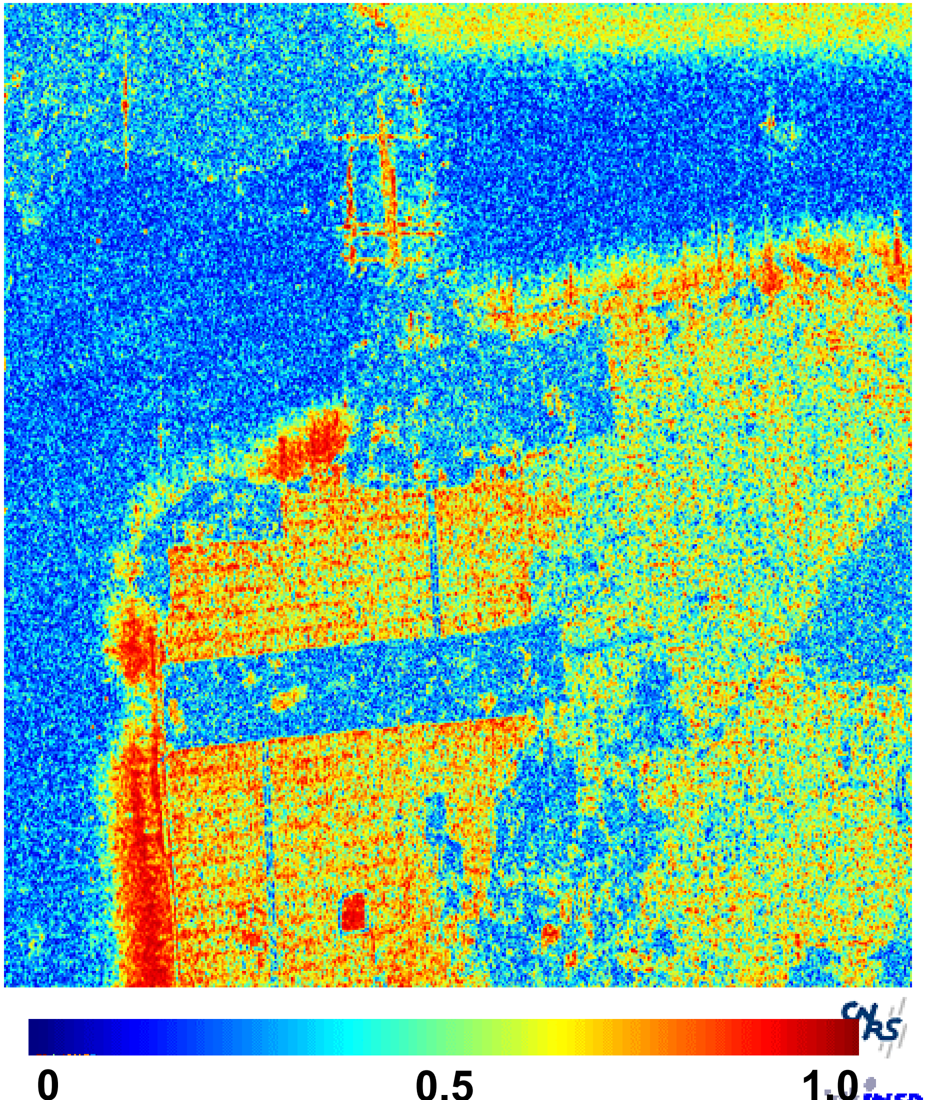


ANISOTROPY (A)

H / A / α DECOMPOSITION



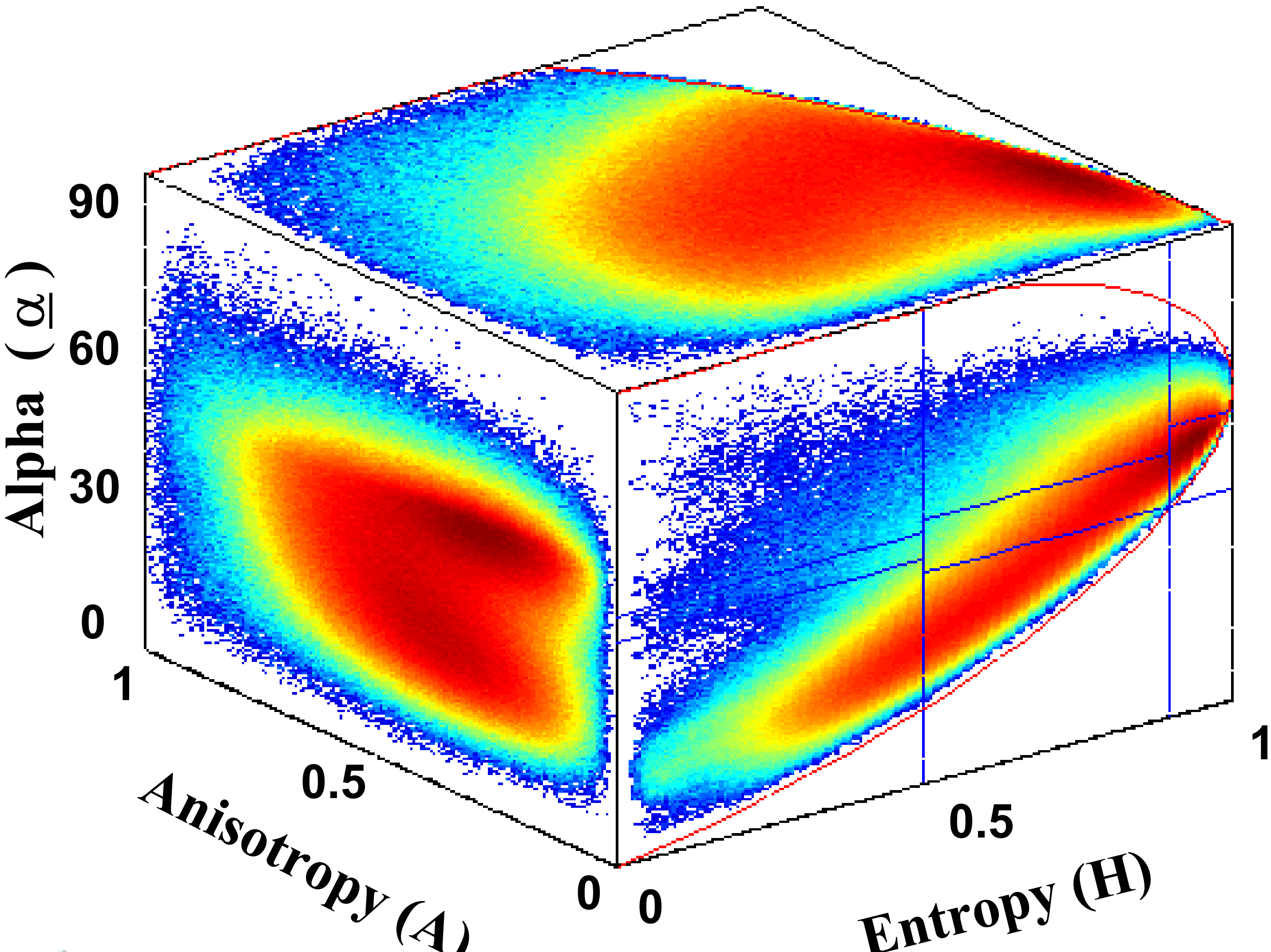
ENTROPY (H)



ANISOTROPY (A)

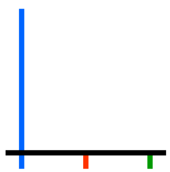
H / A / α DECOMPOSITION

POLSAR DATA DISTRIBUTION IN THE H / A / α SPACE



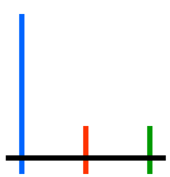
H / A / α DECOMPOSITION

(1-H)(1-A)

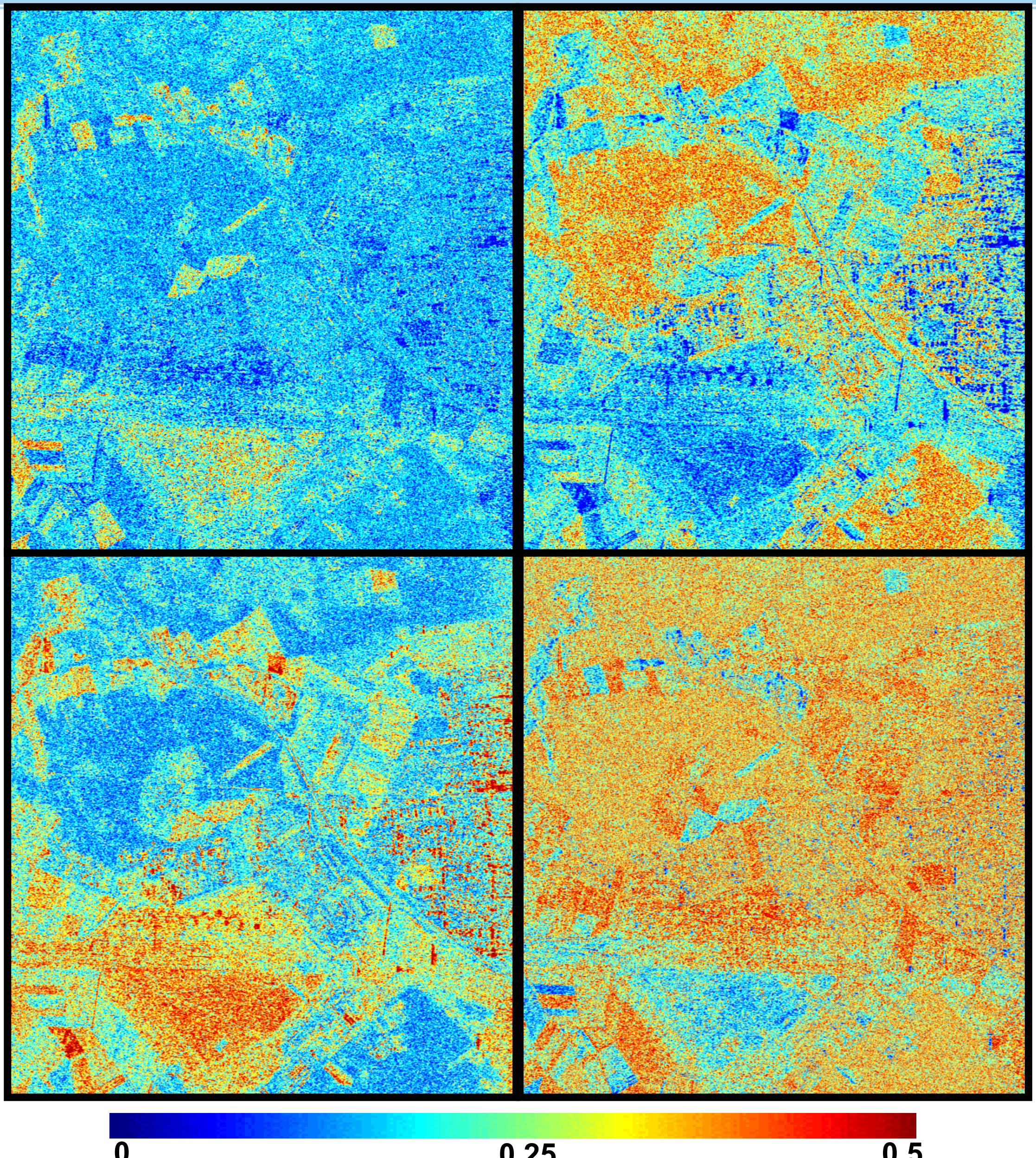


1 MECHANISM

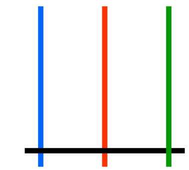
A(1-H)



2 MECHANISMS

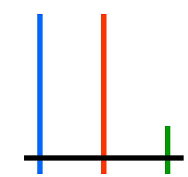


H(1-A)



3 MECHANISMS

HA



2 MECHANISMS

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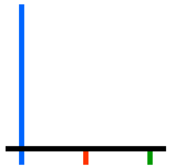
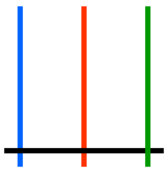
H / A / α DECOMPOSITION

(1-H)(1-A)

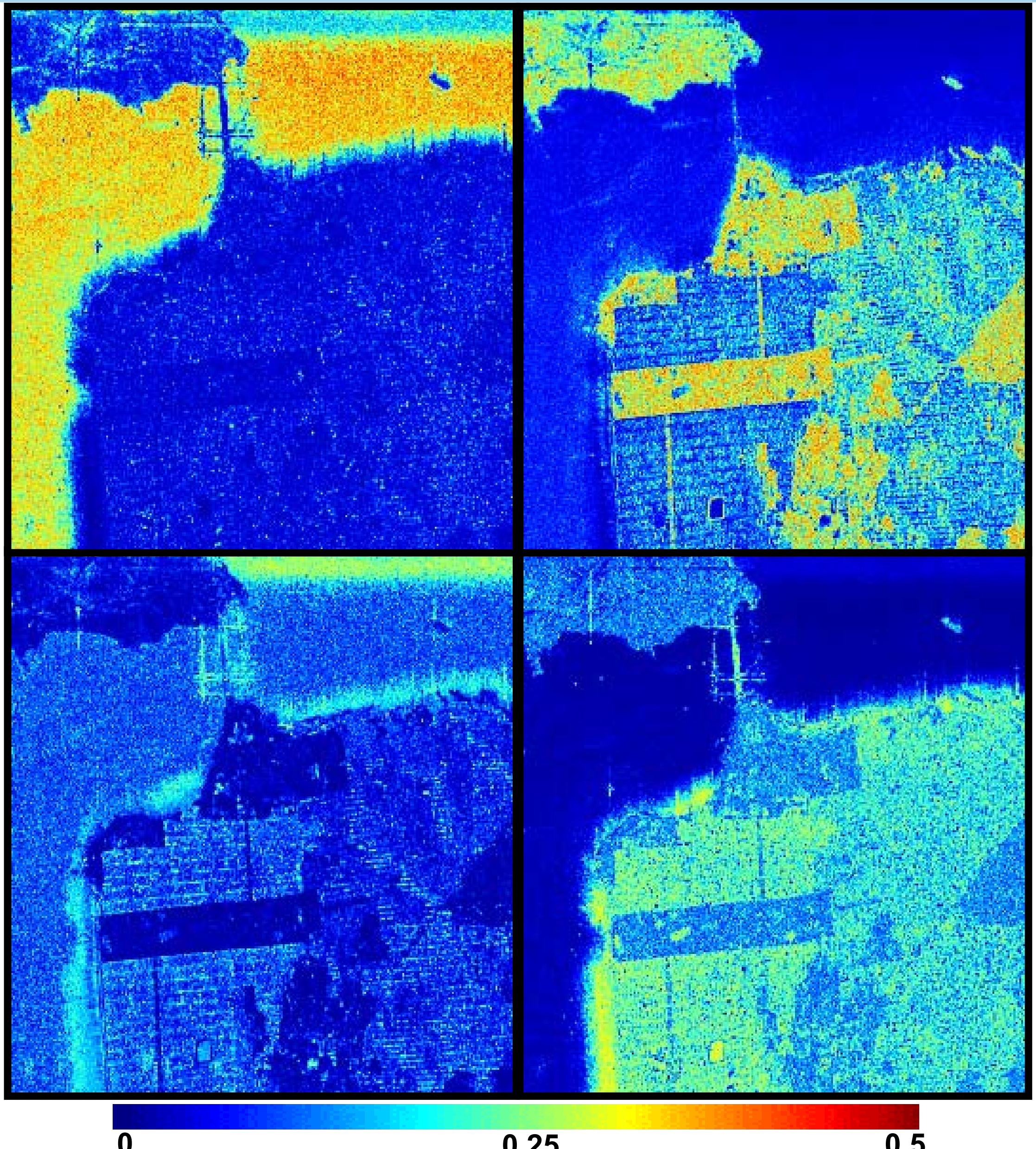


1 MECHANISM

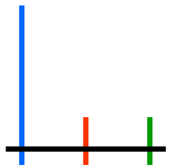
H(1-A)



3 MECHANISMS

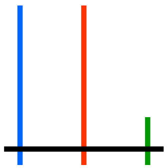


A(1-H)



2 MECHANISMS

HA



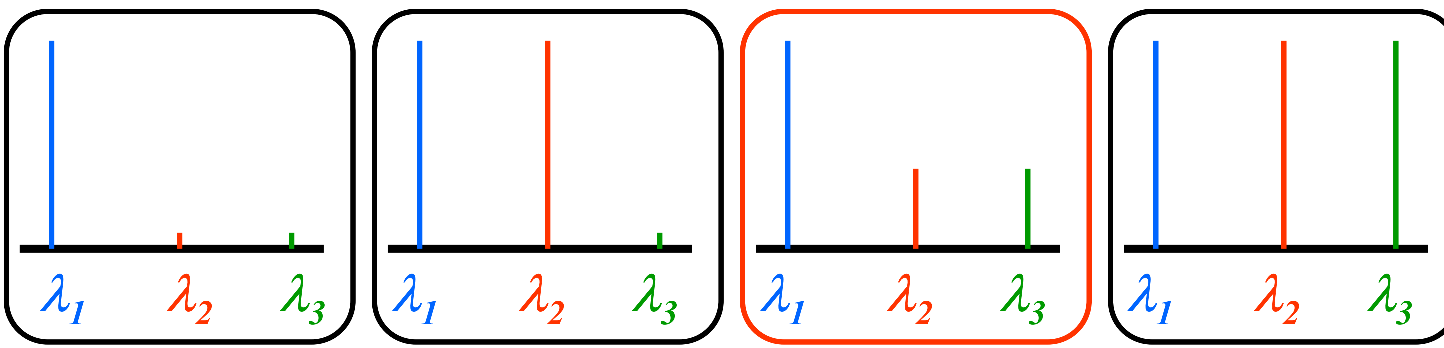
2 MECHANISMS

H / A / α DECOMPOSITION

SCATTERED POWER

$$P = P.*(1\text{-}H)(1\text{-}A) + P.*HA + P.*(1\text{-}H)A + P.*H(1\text{-}A)$$

ONE MECANISM	TWO MECANISMS	TWO / THREE MECANISMS	THREE MECANISMS



H / A / $\underline{\alpha}$ DECOMPOSITION

ENTROPY

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

$\underline{\alpha}$ PARAMETER

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$

ANISOTROPY

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

3 ROLL INVARIANT PARAMETERS

$$\underline{I} = \begin{bmatrix} \underline{\alpha} \\ HA \\ H(1-A) \\ (1-H)A \\ (1-H)(1-A) \end{bmatrix}$$

→ PHYSICAL SCATTERING MECHANISM

→ TYPE OF SCATTERING PROCESS

SEGMENTATION / CLASSIFICATION